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FOREWORD

These Proceedings contain selected papers from those submitted by a part of mathematicians lecturing at the 8th Conference on Analytic Functions held in Poland at Błażejewko (Lake District, Province of Poznań) during the eight days from August 19 to 27, 1982. These papers form the extended versions of their lectures.

According to the tradition of the preceding seven conferences (held in Łódź 1954, Lublin 1958, Kraków 1962, Łódź 1966, Lublin 1970, Kraków 1974, and Kozubník 1979) the topics chosen are rather homogeneous. A considerable part of the papers is concerned with extremal methods and their applications to various branches of complex analysis: one and several complex variables, quasiconformal mappings and complex manifolds. This is however not a rule and the organizers decided to accept also papers on other subjects in complex analysis if they were of good quality.

The Organizing Committee consisted of: C. Andreian-Cazacu (Bucharest), Z. Charzyński (Łódź), P. Dolbeault (Paris), J. Eells (Coventry), A.A. Gončar (Moscow), J. Górski (Katowice), H. Grauert (Göttingen), L. Iliev (Sofia), S. Kobayashi (Berkeley), J. Krzyż (Lublin), O. Lehto (Helsinki), P. Lelong (Paris), J. Ławrynowicz (Łódź) - Chairman, S.N. Mergeljan (Erevan), J. Siciak (Kraków), W. Tutschke (Halle/Saale), and A. Marciniak (Łódź) - Secretary. The Conference was attended by 108 participants (70 from Poland) representing 14 countries.

The Conference was sponsored and organized by the Institute of Mathematics of the Polish Academy of Sciences in collaboration with the Institute of Mathematics of the Łódź University.

The Organizing Committee of the Conference expresses its gratitude to the Springer-Verlag for kind consent of publishing for the second time the Proceedings in the series "Lecture Notes in Mathematics".

Łódź, May 1983 Julian Ławrynowicz

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CONDITION OF CONFORMAL RIGIDITY OF HYPERBOLIC MANIFOLDS WITH BOUNDARIES

Boris Nikolaevič Apanasov (Novosibirsk)

Contents

1. Introduction

Let M be an n-dimensional manifold with boundary ∂M, such that int M is a hyperbolic manifold (of infinite volume), and let there be assigned a quasiconformal mapping f on a similar manifold M' whose contraction upon the boundary ∂M is conformal. Will the manifolds M and M' be isometric ? This problem generalizing the rigidity problem of hyperbolic manifolds without boundary (see [1-5]), was set up in the framework of the theory of deformations of Kleinian groups by Bers [6] and Kruškal [7-9]: will Kleinian groups on n-dimensional sphere S^n conjugate in the Möbius group M_n if they are conjugated by a quasiconformal homeomorphism f of the sphere, f being conformal on a discontinuity set ? The author has shown [10,11] that without imposing additional constrains this problem has a negative solution; besides, from the proposed proof it is clear that one cannot remove the quasiconformality condition of the cojugating mapping f (or, rather, the condition of keeping the measure on S^n by the mapping f).

For a Riemannian manifold X and a fixed point $p \in X$ we denote by X(r) its submanifold consisting of points removed from p for the distance $\leq r$, and by V(r,n) denote the volume of a ball of radius r in n-dimensional hyperbolic space. Then the main result of the paper can be formulated as (cf. [12,15]):

THEOREM A. Let M and M' be n-dimensional manifolds with boundaries, whose interiors are hyperbolic manifolds; $f: M \to M'$ is a quasi-conformal mapping, conformal on the boundary ∂M; $X_s \subset M$ is the s-neighbourhood of the minimal convex retract X of the manifold M. If for some $s > 0$

$$(1.1) \quad \lim_{r\to\infty}[\mathrm{Vol}\, X_s(r)/V(r,n)] = 0$$

then manifolds M and M' are isometric.

The main points of the proof of this theorem are the description of ergodic properties of discrete Möbius groups by Sullivan [5] (see Section 3) and the ideas close to [13] and Ch. 6 of [15].

2. Convex retracts in manifolds and the geometry of fundamental polyhedra

If on some M^n a hyperbolic structure is introduced, then we denote by G the image of the fundamental group $\pi_1(M^n)$ when mapping the holonomy H

$$(2.1) \quad H : \pi_1(M^n) \to G \subset \mathrm{Isom}\, H^n = M_{n-1}.$$

The group G is a discrete group of hyperbolic isometries. But if M^n is also the interior of some manifold with boundary, then G acts discontinuously on the sphere $S^n = \partial H^n$, i.e. it is the Kleinian group on S^n. Then manifold M^n is restored by factorizing the space H^n by the group G.

D e f i n i t i o n 2.1. A convex (in hyperbolic geometry) domain of the limit set $L(G)$ of the group G, i.e. the set

$$(2.2) \quad H_G = \cap(Q \subset \overline{H^n} : L(G) \subset Q \text{ and } Q \text{ is convex})$$

is called the convex Nielsen domain of the discrete group $G \subset \mathrm{Isom}\, H^n$.

Except the groups which are the continuation of Fuchsian groups from R^{n-1} and have, as a Nielsen domain, a subset on some $(n-1)$-dimensional hyperbolic plane, the Nielsen domain ∂H_G of a discrete group G in H^n has dimension n. Its boundary ∂H_G consists of geodesics whose infinitely removed ends are the ends of Euclidean intervals in R^{n-1} lying in the discontinuity set $O(G)$. Therefore, ∂H_G is developed in a hyperbolic plane of dimension $\leq n-1$. In terms of Riemannian geometry the outer curvature of ∂H_G equals 0, and the inner curvature (sectional curvature of the space H^n) equals -1.

The group G leaves the domain H_G invariant and on its subset $H_G - L(G)$ acts discontinuously. The space of orbits of the group G on this set is the convex hyperbolic manifold denoted by M_G:

(2.3) $M_G = [H_G - L(G)]/G$.

If the group G acts on $\partial H^n = S^n$ discontinuously, then the manifold M_G has boundary on which the hyperbolic structure is introduced; this structure is induced by the hyperbolic metric of the space H^n. The manifold M_G is a natural retract of the manifold $M(G) = (\overline{H^n} - L(G))/G$ whose interior is a hyperbolic manifold. This retraction $r : M(G) \to M_G$ is induced by the retraction

(2.4) $\bar{r} : \overline{H^n} \to H_G$

defined as follows:

(2.5) $\bar{r}(x) = x$ for $x \in H_G$, $\bar{r}(x) = x_o$ for $x \in \overline{H^n} - H_G$.

Here x_o is the nearest point to x from the domain H_G in the case where $x \in H^n - H_G$; but if the point x is taken from $\partial H^n - L(G)$, then as the nearest point x_o one takes the first point of contact with H_G of the horosphere with the centre in the point x. This definition is correct due to strict convexity in the hyperbolic geometry of a ball and a horoball.

Note that the limit set is the minimal closed set on sphere S^{n-1}, which is invariant with respect to the action of the group G. Hence, the manifold M_G is the minimal convex retract of the manifold $M(G)$.

By a <u>convex fundamental polyhedron</u> P of the discrete group $G \subset \mathrm{Isom}\, H^n$ we mean a polyhedron with the following properties:

1. P is an open domain in H^n; it is the intersection of no more than a countable family of hyperbolic half-spaces Q_i with boundary planes S_i; the intersection $\bar{P} \cap S_i$ is said to be a <u>side</u> of P.

2. Every compact set in H^n only intersects a finite number of sides of P (the boundary P in H^n only consists of sides).

3. P does not contain G-equivalent points and the images of its closure cover the whole discontinuity set in $\overline{H^n}$.

4. Sides of P are identified pairwise by elements of the group.

5. Every point in H^n has a neigbourhood which only intersects a finite number of images $g(P)$, $g \in G$.

The latter condition (the property of the local finiteness) in dimension $n \geq 3$ does not follow from the former ones. This was shown by Tetenov [14]. He also obtained sufficient conditions of local finiteness. In particular, the Dirichlet polyhedron is of such a kind.

Also the aforesaid about the minimal convex retract of the manifold M can be formulated as follows:

LEMMA 2.2. The minimal convex retract of the manifold M, whose interior is a hyperbolic manifold, is obtained by identifying G-equivalent sides of the polyhedron

$$(2.6) \quad P_H = H_G \cap [\bar{P} - L(G)],$$

where $P \subset H^n$ is a convex fundamental polyhedron of the group $G = H[\pi_1(M)]$ of hyperbolic isometries.

3. Ergodic properties of discrete Möbius groups

In this section we describe some results of Sullivan [5] we need further.

The action of the discrete group $G \subset \mathrm{Isom}\, H^n$ in the sphere $S^{n-1} = \partial H^n$ to which the set of zero measure is divided into two parts - dissipative and conservative. The dissipative part is the union of pairwise intersecting measurable sets represented by the elements of G (the analogy of the action in H^n). The conservative part K is characterized by the fact that for any subset $Y \subset K$, $m_{n-1}(Y) > 0$, there exists a sequence of distinct elements $g_i \in G$ such that for all the numbers i

$$m_{n-1}[Y \cap g_i(Y)] > 0.$$

D e f i n i t i o n 3.1. The point $s \in S^{n-1}$ is called a horospherical limit point of the group $G \subset \mathrm{Isom}\, H^n$, if the orbit $G(p)$ of some fixed point $p \in H^n$ enters any horosphere with the centre at the point s. We denote the set of horospherical limit points of group G by $L_h = L_h(G)$.

For the points $s \in S^{n-1} - L_h$ we increase the radius of the horosphere with the centre at s till we come across some point x from the orbit $G(p)$. If such a point x of the orbit is also unique we call it the nearest to s point of the orbit.

THEOREM 3.2. For any discrete group $G \subset \mathrm{Isom}\, H^n$ and for any choice of the orbit $G(p) \subset H^n$, the union $L_h(G)$ with a set of points on the sphere ∂H^n having the nearest points of the orbit is a full measure set. Moreover, this division of the sphere is the division of the action of G on the conservative $(= L_h(G))$ and dissipative parts.

COROLLARY 3.3. Let $P \subset H^n$ be a convex fundamental polyhedron of the discrete group $G \subset \mathrm{Isom}\, H^n$. Then the dissipative part of the action of G upon ∂H^n is the set

$$\bigcup_{g \in G} g[\partial H^n \cap \bar{P}]$$

From Corollary 3.3 and Theorem 3.2 itself, there follows directly the discription of the action of G on its limit set (cf. [15]).

COROLLARY 3.4. The group G acts conservatively on the set $L(G)$ $(m_{n-1}[L(G)] > 0)$ iff

(3.1) $m_{n-1}[L(G) \cap \bar{P}] = 0.$

4. Proof of Theorem A

Let $P \subset H^n$ be a convex fundamental polyhedron of the group $G = H[\pi_1(M)] \subset \mathrm{Isom}\, H^n$, and $\bar{P}$ be its closure in $\overline{R^n}$ (we assume that H^n is the Poincaré model in the half-space). Firstly, let us prove that the condition (1.1) of the theorem is equaivalent to the condition (3.1). Let $P^* = L(G) \cap \bar{P}$. Suppose that $m_{n-1}(P^*) > 0$. Then almost all points $x \in P^*$ are the density points for P^*, i.e. they are characterized by the fact that

(4.1) $\lim_{r \to 0} \{m_{n-1}[B^{n-1}(x,r) \cap P^*]/m_{n-1}[B^{n-1}(x,r)]\} = 1.$

If now we fix in the polyhedron P_H from Lemma 2.2 the point x_0 corresponding to the initial point p in the minimal convex retract $(= M_G)$, then the spherical measure of the set P^* is the solid angle at which this set is seen from the point x_0. Hence, by (4.1), the limit in (1.1) tends to this measure which we assume to be positive.

Conversely, if $m_{n-1}(P^*) = 0$, then consider the sphere S_r of the radius $r > 0$ with the centre at x_0 from the polyhedron P_H. Denote by $w(r)$ the solid angle of the part of S_r which lies in the s-neighbourhood of the polyhedron P_H and intersects the polyhedron P. From the convexity of P_H, since the measure of its limit vertices is zero, it follows that with increasing the radius r the value of $w(r)$ decreases to zero. If $a(r)$ is the volume of $(n-1)$-dimensional sphere with radius r, then

$$(4.2) \quad \frac{\mathrm{Vol}\, X_s(r)}{V(r,n)} = \frac{\mathrm{Vol}(M_G)_s(r)}{V(r,n)} = \int_0^r a(r)w(r)\,dr / \int_0^r a(r)\,dr.$$

Observe that for any $t>0$ there exists an r_0 such that the angle $w(r)$ is less than t for $r > r_0$. Summing up, we obtain

$$(4.3) \quad \frac{\mathrm{Vol}\, X_s(r)}{V(r,n)} = \frac{\int_0^{r_0} a(r)w(r)\,dr}{\int_0^r a(r)\,dr} + t\,\frac{\int_{r_0}^r a(r)\,dr}{\int_0^r a(r)\,dr}.$$

The first term on the right-hand side of inequality (4.3) with the increase of r tends to zero, and another one is not greater than any arbitrarily chosen $t>0$. This proves that relation (1.1) holds.

Using Corollary 3.3 we conclude that the condition (1.1) is equivalent to the conservativeness of the action of G upon its limit set.

Yet, on the conservative part of the action of G on the sphere ∂H^n there does not exist a measurable tangent field of k-dimensional planes, $1 \le k \le n-1$, G-invariant almost everywhere. This fact, proved by Sullivan for the planar case [5], takes place for any n. Its complete proof can be found in [15]. Our version of the proof gives some improvement since it applies to any dimension. A different proof can be found in [16].

Hence, it follows that if W is a measurable conformal structure on the sphere ∂H^n (in tangent space), invariant a.e. relative to the group G, then W a.e. coincides with the standard conformal structure on the conservative part of the action of G on the sphere ∂H^n, i.e. on the set L(G). This follows from the fact that when comparing the structure W on ∂H^n to the standard conformal structure, there arises a.e. a field of ellipsoids determined up to the dilatation. Thus, it is proved that the mapping f conjugating the groups G and G' on the limit set a.e. has a distortion coefficeint equal to 1 as well, i.e. it is conformal (Möbius), and the groups G and G' are conjugated in Isom H^n. Hence, it follows that the manifolds M and M' corresponding to the groups are isometric.

Remark 4.1. The condition of Theorem A is sufficient but not necessary. This is shown by the example of the functional group on the sphere S^n, $n \geq 3$, having the limit set of the positive measure (it is constructed similarly to the example of Abikoff [17] who uses the Peano curve). In this group a set of limit vertices of a convex fundamental polyhedron in $H^{n+1} = \text{int } S^n$ has the full measure m_n of "the Peano surface", which is positive by construction. At the same time, as it follows from the result of Kruškal [8] (Theorem 4, actually proved for functional groups only), this group is rigid in the above sense.

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ON CARLEMAN APPROXIMATION BY MEROMORPHIC FUNCTIONS

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I. CONDITIONS K AND G

Let E be a relatively closed subset of a domain D in the complex plane (closed in the D-topology). We denote by H(D) the functions holomorphic in D and by A(E) the functions continuous on E and holomorphic in the interior E^o of E . If for every pair of functions $\{f(z),\varepsilon(z)\}$, $f \in A(E)$ and $\varepsilon(z)$ positive and continuous on E , there exists a function $g \in H(D)$ such that

$$|f(z)-g(z)| < \varepsilon(z) \ , \ z \in E$$

then E is called a Carleman set in D . If we restrict ourselves to closed sets E with empty interior $(E^o = \phi)$, then this definition amounts to the definition given by P.M. Gauthier (in this volume). In 1927, T. Carleman [2] proved that the real line $\mathbb{R}$ is a Carleman set in $\mathbb{C}$. See also Kaplan [9], Sinclair [17] and Hoischen [7].

Let $D^* = D \cup \{*\}$ be the one-point compactification of D . We will say that E satisfies condition K if $D^* \setminus E$ is connected, and locally connected at infinity (i.e. locally connected at the point "*"). M. V. Keldysh [10] and A. Roth [15] introduced this condition in connection with problems in approximation. In 1968, N.U. Arakeljan [1] showed that this condition was equivalent to the possibility of uniform approximation of every function continuous on E and holomorphic in the interior E^o by functions holomorphic in all of D , i.e.

$$A(E) = \overline{H(D)}^E \iff \text{condition } K ,$$

where $\overline{H(D)}^E$ denotes the uniform closure (i.e. closure in the sup norm) on E of the space of functions H(D) . Thus, indeed, this condition must be satisfied by all Carleman sets.

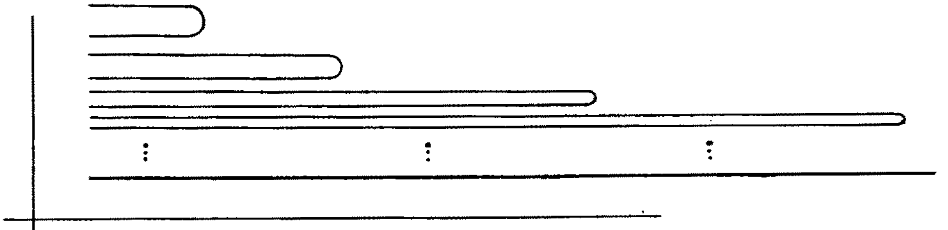
An other condition related to the characterization of Carleman sets was introduced in 1969 by P.M. Gauthier [5] and shown to be necessary. E is said to satisfy condition G , if for every compact $K \subset D$, there exists a compact Q

(depending on K) such that no components of E^o meets both K and $D \setminus Q$.
($D \setminus Q$ can be thought of as a neighborhood of the point "at infinity" *).

II. EXAMPLES

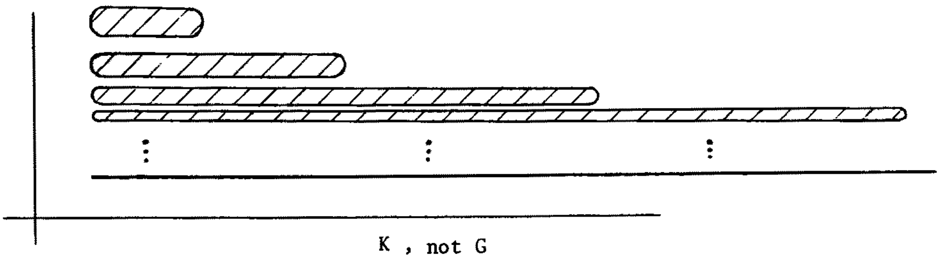
Actually these conditions characterize Carleman sets (see section III), so let us give a few examples. In these examples D will always be the whole complex plane $\mathbb{C}$.

1) long fingers



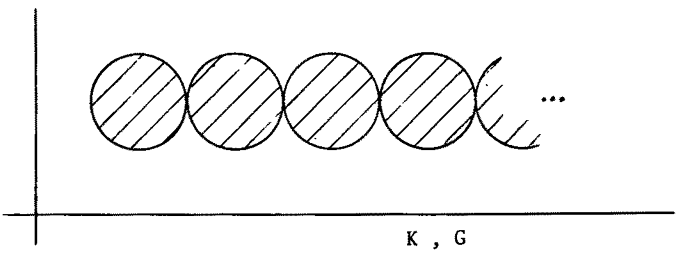
not K , G

2) long islands



K , not G

3) tangent discs



K , G